JANUARY 2003

EXAMINATION

MATHEMATICS

Paper 02 - General Proficiency

2 hours 40 minutes

03 JANUARY 2003 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and any TWO in Section II.
2. Begin the answer for EACH question on a NEW page.
3. Full marks may not be awarded unless full working or explanation is shown with the answer.
4. Mathematical tables and graph paper are provided.

   A List of Formulae is provided on page 2 of this booklet.
5. Mathematical instruments are needed for this paper.
6. Silent electronic calculators may be used for this paper.
7. You are advised to use the first 10 minutes of the examination time to read through this paper. Writing may begin during this 10-minute period.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

LIST OF FORMULAE

Volume of a prism

$V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of a right pyramid

$V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference

$C = 2\pi r$ where r is the radius of the circle.

Area of a circle

$A = \pi r^2$ where r is the radius of the circle.

Area of trapezium

$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations

If $ax^2 + bx + c = 0$,

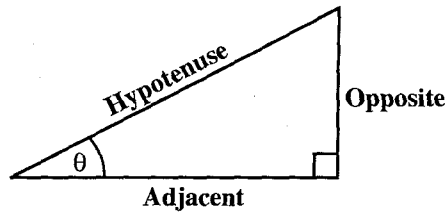
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

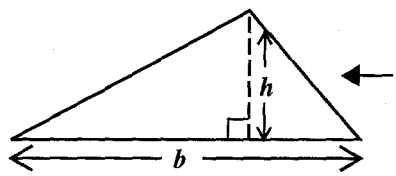
Trigonometric ratios

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$





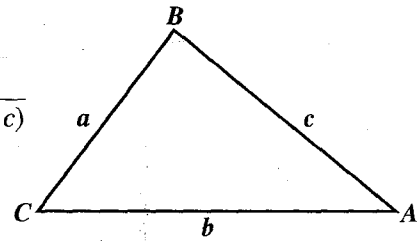
Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a + b + c}{2}$



Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Calculate the exact value of

$$\frac{3\frac{1}{2} + 1\frac{5}{6}}{1\frac{3}{5}}$$

giving the answer as a fraction in its LOWEST terms. **(4 marks)**

(b) Write 2684 correct to 2 significant figures. **(1 mark)**

(c) A man deposits \$800 in his account at a bank which offers 6% simple interest per annum.

(i) How much interest would he receive on the \$800 after 9 months?

(ii) How long would it take for the \$800 to increase to \$992? **(5 marks)**

Total 10 marks

2. (a) Solve for x,

$$\frac{x+2}{2} - \frac{x-1}{3} = 2.$$

(3 marks)

(b) Factorize completely:

(i) $4x^2 - y^2$

(ii) $6m + 4n - 9km - 6kn$

(iii) $2a^2 + a - 6$ **(5 marks)**

(c) Given $s - 3t = rt$,

(i) express t in terms of r and s

(ii) calculate the value of t when $r = 2$ and $s = 15$. **(4 marks)**

Total 12 marks

3. (a) In a group of 55 students,

31 students passed French
$2x$ students passed Spanish only
x students passed both French and Spanish
10 students passed neither French nor Spanish.

(i) Draw a CLEARLY labelled Venn diagram to illustrate the information above.

(ii) Calculate the number of students who passed Spanish.

(iii) Calculate the number of students who passed ONLY ONE subject.

(8 marks)

(b) Given that $P = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$ and $Q\begin{pmatrix} -2 & 2 \\ 1 & 1 \end{pmatrix}$, calculate

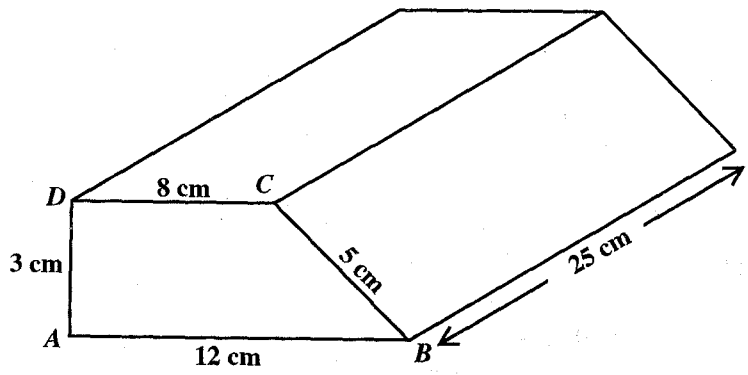
(i) $2P + Q$

(ii) P^2. **(4 marks)**

Total 12 marks

4. (a) A scale of 1:25 000 is used to draw a map of an island.
Calculate the actual distance, in km, between two points on the island if the distance between the points on the map is 36 cm. **(2 marks)**

(b)



The diagram above, **not drawn to scale**, shows a wooden prism of length 25 cm. The cross-section $ABCD$ is a trapezium with AB parallel to DC, $B\hat{A}D = 90°$, $AB = 12$ cm, $BC = 5$ cm, $CD = 8$ cm and $AD = 3$ cm.
Calculate

(i) the area, in cm^2, of the cross-section, $ABCD$

(ii) the volume, in cm^3, of the prism

(iii) the total surface area, in cm^2, of the prism. **(8 marks)**

Total 10 marks

5. (a) (i) **Using ruler and compasses only**, construct **parallelogram *WXYZ*** where $WX = 5.5$ cm, $WZ = 7.0$ cm and $X\hat{W}Z = 60°$.

(ii) Draw diagonal XZ. Measure and state its length. **(6 marks)**

(b) **An answer sheet is provided for this question.**

The diagram on the answer sheet shows triangle LMN, and its image under an enlargement, triangle $L'M'N'$.

(i) State the coordinates of the points L', M' and N'.

(ii) Find C, the centre of enlargement, by using straight lines to join pairs of corresponding vertices of the triangles. State the coordinates of C.

(iii) What is the scale factor of the enlargement? **(6 marks)**

Total 12 marks

6. (a) At a school shop, pens are sold at x dollars each and rulers at y dollars each. Mr James bought 4 pens and 5 rulers for \$24. Mrs Singh bought 2 of the same pens and 7 of the same rulers for \$21.

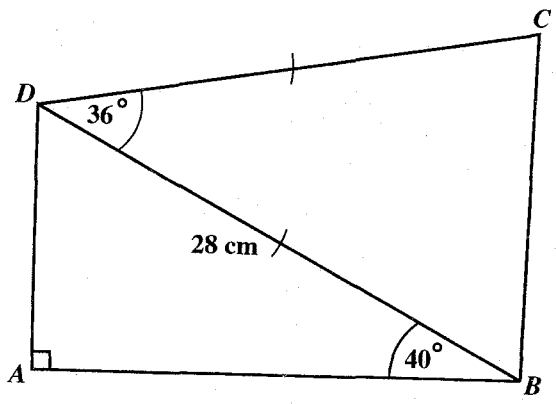
(i) Write TWO equations in x and y to represent the information given above.

(ii) Solve the equations.

(iii) Calculate the TOTAL cost for 1 pen and 1 ruler.

(6 marks)

(b)



In the diagram above, **not drawn to scale**, $BD = CD$, $BD = 28$ cm, $B\hat{A}D = 90°$, $A\hat{B}D = 40°$ and $C\hat{D}B = 36°$.

Calculate, **stating your answer correct to 1 decimal place**,

(i) the length of the side AB

(ii) the area of the triangle BDC. **(5 marks)**

Total 11 marks

7. (a) Given that $g(x) = 6 - x$ and $h(x) = x^3$,

Calculate

(i) $h(-3)$

(ii) $hg(2)$

(iii) $gh(2)$. **(5 marks)**

(b) (i) Given that $y = 2x^2 - x - 6$,
copy and complete the table below.

x	-2	-1	0	1	2	3
y		-3	-6	-5		9

(ii) **Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis**, draw the graph of $y = 2x^2 - x - 6$ for $-2 \leq x \leq 3$.

(iii) **Use your graph** to solve the equation

$2x^2 - x - 6 = -3$. **(7 marks)**

Total 12 marks

8. The table below shows the time, **to the nearest minute**, taken by 50 students to walk to school.

Time taken/min	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30
No. of students	4	7	10	20	6	3

(a) Calculate an estimate of the mean time taken by the students to walk to school. **(4 marks)**

(b) Estimate the probability that a student, selected at random, takes at least 21 minutes to walk to school. **(2 marks)**

(c) Calculate

(i) the upper boundary of the class interval 16 - 20

(ii) the width of the class interval 11 - 15. **(3 marks)**

(d) The time, **to the nearest minute**, taken by each of 2 other students to walk to school is 33 minutes.

(i) Write down the class interval in which this time would lie.

(ii) State the assumption about the class width for your answer in (d) (i) above. **(2 marks)**

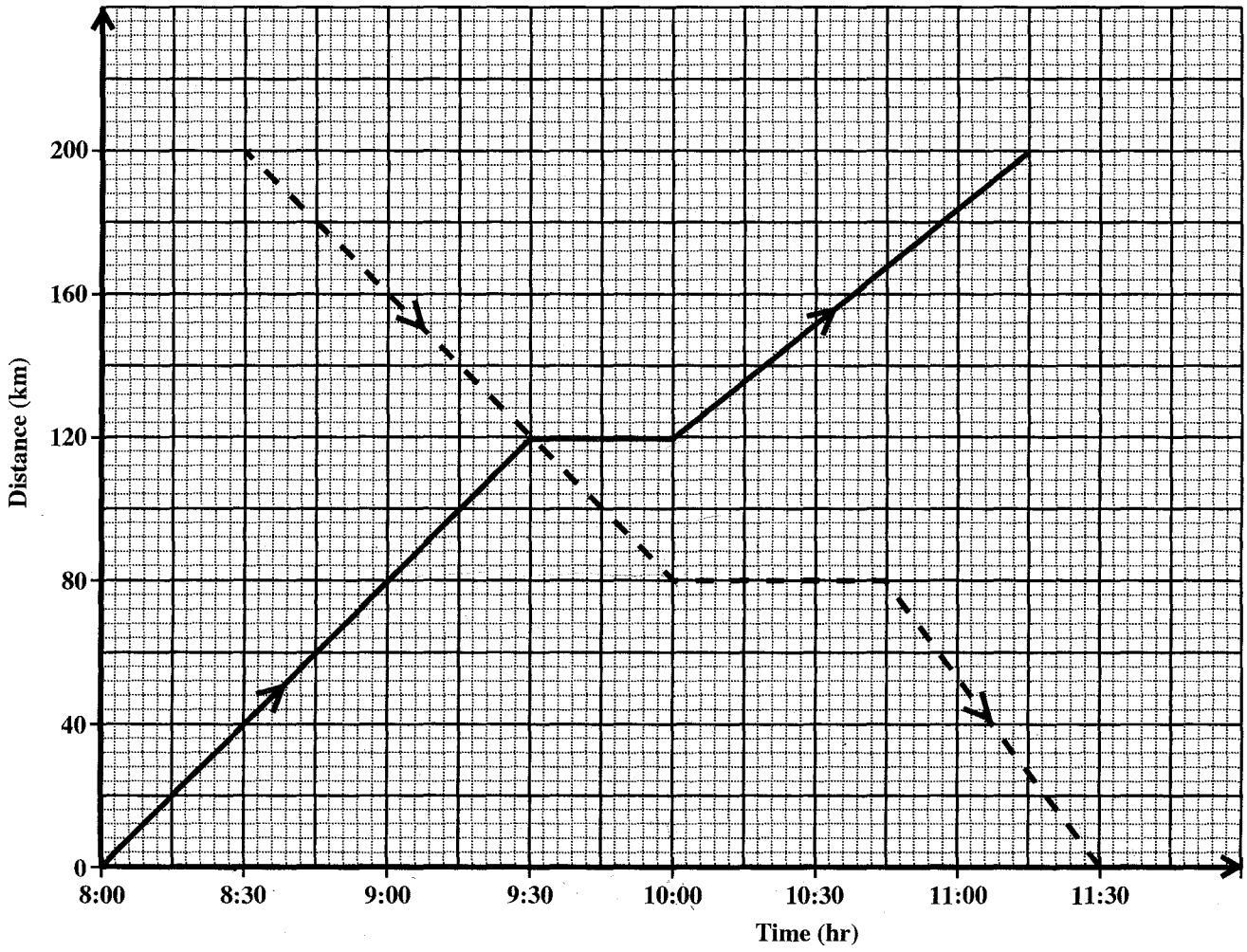
Total 11 marks

SECTION II

Answer TWO questions in this section.

RELATIONS AND FUNCTIONS

9. The distance-time graph below shows the journey of a van and of a jeep. At 8.00 a.m,the van left Town A and travelled to Town B. Later, the jeep left Town B and travelled along the same road, but in opposite direction, to Town A.



(a) **Using the graph**, determine

(i) the distance between Town *A* and Town *B*

(ii) the distance between the two vehicles at 09:00 hours

(iii) the time at which the vehicles meet

(iv) the average speed of the jeep for the entire journey. **(4 marks)**

(b) **Using the graph**, determine EACH of the following for the jeep:

(i) The time it left Town B

(ii) The time it arrived at Town A

(iii) The time at which it first stopped

(iv) The distance travelled before it first stopped

(v) The average speed before the first stop

(vi) The average speed after the first first stop **(6 marks)**

(c) (i) Express $3x^2 - 2x + 1$, in the form $3(x + p)^2 + q$, where p and q are real numbers. State the values of p and q.

Hence, determine for $f(x) = 3x^2 - 2x + 1$,

(ii) the minimum value for $f(x)$

(iii) the value of x for which $f(x)$ is a minimum. **(5 marks)**

Total 15 marks

10. (a) Mr John makes x cakes and y pies each day. In order to supply his customers, he does the following:

(i) He must make AT LEAST 2 cakes each day.
Write an inequality to represent the information.

(ii) He must NOT make FEWER than 3 pies each day.
Write an inequality to represent this information.

(iii) He has NO MORE than \$72 to make the cakes and the pies. Each cake costs \$6 to make and each pie costs \$10 to make.
Write an inequality to represent this information. **(5 marks)**

(b) (i) **Using a scale of 2 cm to represent 1 unit on both axes**, draw the graphs of the inequalities in 10 a (i), (ii) and (iii).

(ii) Write the coordinates of the vertices of the region that satisfies ALL THREE inequalities. **(6 marks)**

(c) Mr John makes a profit of \$8 on each cake and \$12 on each pie.

(i) Write an expression in x and y for the TOTAL profit, P.

(ii) How many cakes and pies must be made in order to obtain the maximum profit?

(iii) Calculate the maximum profit. **(4 marks)**

Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a) For this question, use $\pi = 3.14$ and radius of the earth = 6400 km.

The coordinates of the points L, M and N on the earth's surface are (60°N, 30°W), (12°S, 30°W) and (60°N, 10°W) respectively.

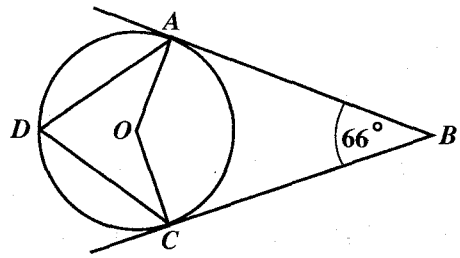
Calculate, to the NEAREST kilometre,

(i) the distance LM, measured along the common circle of longitude

(ii) the distance LN measured along the common circle of latitude.

(10 marks)

Use the diagram below to answer Part (b) of this question.



(b) In the diagram above, **not drawn to scale**, AB and BC are tangents to the circle, centre O. The radius of the circle is 9 cm and angle $ABC = 66°$.

Calculate, **giving reasons for each step of your answer**, the size of EACH of the following angles:

(i) $\angle AOC$

(ii) $\angle ADC$. **(5 marks)**

Total 15 marks

12. Three cricketers, A, B and C are positioned on a cricket ground so that:

A, the bowler is 22 metres south of B, the batsman.
C, the fielder is on a bearing of 205° from B.
The distance between C and B along a straight line is 36 metres.

(a) Draw a diagram showing the positions of A, B and C.

On your diagram show

(i) the north direction

(ii) the points A, B and C

(iii) the distances 22 m and 36 m

(iv) the bearing of 205°. **(4 marks)**

(b) For the diagram in Part (a):

(i) State the size of the angle CBA.

(ii) Calculate, to the NEAREST metre, the distance AC.

(iii) Calculate, to the NEAREST degree, the bearing of A from C.

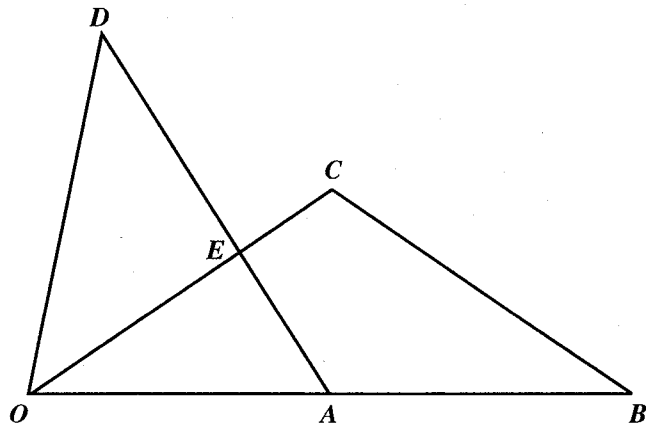
(iv) How far east must C move if he is to be in line with A and B?

(11 marks)

Total 15 marks

VECTORS AND MATRICES

13.



In the diagram above, **not drawn to scale**, A is the midpoint of OB.

$\overrightarrow{EC} = \frac{1}{2}\overrightarrow{OE}$

$\overrightarrow{ED} = 2\overrightarrow{AE}$.

The vectors **u** and **v** are such that

$\overrightarrow{OE} = 2\mathbf{u}$ and $\overrightarrow{OA} = \mathbf{v}$.

(a) Write in terms of **u** and **v**:

(i) $\overrightarrow{AE}$

(ii) $\overrightarrow{AD}$

(iii) $\overrightarrow{BC}$

(iv) $\overrightarrow{CD}$ **(11 marks)**

(b) Using a vector method, show that the points B, C and D lie on a straight line. **(4 marks)**

Total 15 marks

14. (a) M is the matrix $\begin{pmatrix} 3 & 4 \\ p & q \end{pmatrix}$.

If M is a singular matrix and $p = 2$, calculate the value of q. **(2 marks)**

(b) $A = (3 \quad n)$ and $B = \begin{pmatrix} m & 2 \\ 4 & 3 \end{pmatrix}$.

Given that $AB = (11 \quad 3)$, calculate the values of m and n. **(5 marks)**

(c) The matrix, K, maps the point $S(1, 4)$ onto $S'(-4, -1)$ and the point $T(3, 5)$ onto $T'(-5, -3)$.

Given that $K = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

(i) Write down TWO equations in a and b.

(ii) Write down TWO equations in c and d.

(iii) Calculate the values of a, b, c and d.

(iv) Describe the geometric transformation which is represented by the matrix, K.
(8 marks)

Total 15 marks

END OF TEST

JANUARY 2003

EXAMINATION

MATHEMATICS

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Answer sheet for Question 5 (b). **Candidate number.....**